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Signature:
Student ID \#: $\qquad$

- You are allowed a Ti-30x IIS Calculator and one $8.5 \times 11$ inch paper with handwritten notes on both sides. Other calculators, electronic devices (e.g. cell phones, laptops, etc.), notes, and books are not allowed.
- Some questions require you to explain answers: no explanation, no credit.
- Try to show your work on all questions: no work, no partial credit.
- You may use the back of the exam for scratch work: please submit any additional paper you use.
- Place a box around your answer to each question.
- Raise your hand if you have a question.

| 1 | $/ 10$ |
| :---: | :---: |
| 2 | $/ 10$ |
| 3 | $/ 10$ |
| 4 | $/ 10$ |
| 5 | $/ 10$ |
| T | $/ 50$ |
| Good Luck! |  |

(1) Determine bases for the rowspace (3pts), nullspace ( 3 pts ), and column space (3pts) of $A=\left(\begin{array}{ccr}2 & 3 & 5 \\ 8 & 13 & -4\end{array}\right)$.
(1pt) What is the rank of $A$ ?
(2) Consider the matrix $A=\left(\begin{array}{ccc}\sqrt{2} / 2 & 0 & -\sqrt{2} / 2 \\ 3 & 1 & 1 \\ \sqrt{2} / 2 & 0 & \sqrt{2} / 2\end{array}\right)$.
(a) (5pts) Calculate $\operatorname{det}(A)$
(b) (5pts) Determine whether or not $A^{-1}$ exists, and if so compute it.
(3) Consider the matrix

$$
R=\frac{1}{3}\left(\begin{array}{ccc}
1 & 2 & 2 \\
2 & -2 & 1 \\
2 & 1 & -2
\end{array}\right)
$$

(a) (5pts) Calculate the matrix product $R^{T} R$.
(b) ( 5 pts ) What is the general solution to the following $3 \times 3$ linear system?

$$
R\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right)=\left(\begin{array}{l}
1 \\
1 \\
1
\end{array}\right)
$$

(4) (10pts) Let $R$ be the matrix from the Problem 3. Find a nonzero vector $v \in \mathbb{R}^{3}$ such that $R v=v$.
(5) Let $X=\left\{(x, y) \in \mathbb{R}^{2}|y=|x|\right.$ and $x \geq 0\}$ (here $|\bullet|$ denotes the usual absolute value of a real number $\bullet$.)
a) (5pts) Is $X$ a subspace of $\mathbb{R}^{2}$ ? Explain.
b) (5pts) What is the smallest subspace of $\mathbb{R}^{2}$ containing $X$ ?

